Course: BMG2CC1B

By Sudipto Bhattacharjee

Phone No: 9476296853 (Whatsapp no).

Ex 1: Eliminate the arbitrary functions f and F from:

$$y = f(x-at) + F(x+at)$$

Sol: $y = f(x-at) + F(x+at)$

$$\frac{\partial y}{\partial x} = f'(x-at) + F'(x+at)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(x-at) + F''(x+at)$$

$$\frac{\partial y}{\partial t} = -a\, f'(x-at) + F'(x+at) \cdot a$$

$$\frac{\partial^2 y}{\partial t^2} = a^2 \{ f''(x-at) + F''(x+at) \} = \frac{\partial^2 y}{\partial x^2} a^2$$

Hence $\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = 0$ is the required PDE

Lagrange's Method for solving PDE

An PDE of the form $Pp + Qq = R$ where P, Q, R are function of x, y, z is known as Lagrange's first order PDE.

Step I: Transform the given PDE of first order in standard form.

$$Pp + Qq = R$$

Step II: Write down Lagrange's Auxiliary Equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Step III : Solve using known methods,

Let $u(x,y,z) = c_1$ and $v(x,y,z) = c_2$ be two independent solutions

Step IV The General Solution can be written as $\phi(u,v) = 0$ or $u = \phi(v)$ or $v = \phi(u)$

Example: Solve $a(p+q) = z$

So Lagranger Auxiliary Equation is

$$\frac{dx}{a} = \frac{dy}{a} = \frac{dz}{1} \quad -(1)$$

Hence from (1) $\frac{dx}{a} = \frac{dy}{a} \Rightarrow dx - dy = 0$

$\Rightarrow x - y = c_1$ (integrating)

where c_1 is integrating constant.

Also from (1.)

$\frac{dy}{a} = \frac{dz}{1} \Rightarrow dy - a\,dz = 0$

$\Rightarrow y - az = c_2$ (by integration)

c_2 is an integrating constant.

So general solution..

$$\phi(x-y,\ y-az) = 0$$

where ϕ is an arbitrary function.

Scanned with CamScanner